NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 4096

FLOW-TURNING LOSSES ASSOCIATED WITH ZERO-DRAG EXTERNAL-COMPRESSION SUPERSONIC INLETS

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SUMMARY

An analysis based on momentum and continuity considerations is used to evaluate the total-pressure recovery of zero-wave-drag external-compression supersonic air inlets for the Mach number range from 1.0 to 4.0.

The geometry of such inlets can cause a significant loss in inlet total-pressure recovery which arises in the process of turning the flow back to the axial direction after supersonic compression. This loss may become as large as 20 percent of the computed inlet recovery at Mach 4.0.

Some consideration is given to wind tunnel blockage calculations in which the model drag enters as a parameter, and a criterion is developed which supplements the usual Kantrowitz condition.

INTRODUCTION

The design of a supersonic air inlet for a propulsion system usually requires many compromises to obtain satisfactory over-all performance. One such compromise involves the interplay of inlet total-pressure recovery and cowl pressure drag. Large cowl pressure drags are usually associated with high-performance external-compression inlets (ref. 1, p. 627). Drag reduction has been obtained primarily by decreasing the cowl projected area, that is, by turning the inlet flow back to the axial direction more rapidly.

The limiting form of such a philosophy is the zero-wave-drag inlet in which the external cowling is alined with the free-stream velocity. The recovery potential of this type of inlet is necessarily less than that of a conventional inlet because of the limited supersonic compression permitted by the requirement of internally attached shocks at the cowl lip. Further total-pressure losses are also incurred by abruptly turning the inlet flow back to the axial direction.

An analysis based on momentum and continuity considerations whereby these latter losses may be calculated is presented along with summary curves applicable to zero-drag inlets with a sharp centerbody shoulder. The effect of rounding the centerbody shoulder is also considered as well as some aspects of permissible inlet contraction and the analogous problem of wind tunnel blockage.

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SYMBOLS

A	area
D(M)	$M\left(1 + \frac{\gamma - 1}{2} M^2\right)^{-\frac{\gamma+1}{2(\gamma-1)}}$
dD(M)/dN(M)	$G(M)(1 + \gamma M^2)$
G(M)	$(1 + \gamma M^2)\left(1 + \frac{\gamma - 1}{2} M^2\right)^{-\frac{\gamma}{\gamma-1}} = (1 + \gamma M^2)\left(\frac{p}{P}\right)_M$
$\hat{i}$	unit vector
M	Mach number
M_1^*	Mach number after Kantrowitz contraction from M_1
N(M)	D(M)/G(M)
$\hat{n}$	unit outward normal vector
P	total pressure
p	static pressure
q	dynamic pressure, $q = \frac{1}{2} \rho V^2 = \frac{\gamma}{2} pM^2$
R	total-pressure recovery, $R = P_2/P_0$
S	bounding surface of integration
V	velocity
$\bar{V}$	velocity vector
Z	force parameter, eq. (15)
α	flow angularity at inlet entrance station

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γ	ratio of specific heats for air, 1.4
η	cowl-lip overhang angle, deg
θ	cone half-angle, deg
ρ	mass density
φ	$\frac{p_R^m}{P_0}\frac{A_R}{A_0} - \frac{p_c^m}{P_0}\frac{A_c}{A_0}$

Subscripts:

c	cowl
i	basic inlet
R	ramp or centerbody
s	shoulder
0	free stream
1	inlet entrance station
2	equivalent uniform flow
$()_M$	evaluated at M

Superscripts:

m	average value
*	corresponding to Kantrowitz contraction

ANALYSIS

The momentum equation for the steady flow of a compressible fluid for a given control volume may be written in vector form as

$$\oint_S (\rho\overline{V} \cdot \hat{n})\overline{V}\, dA + \oint_S p\hat{n}\, dA = 0 \tag{1}$$

if boundary shearing stresses are neglected. The notation $\oint_S$ indicates the integration is to be performed over the entire surface S enclosing the control volume, while $\hat{n}$ is a unit outward normal vector to the surface S. The first integral in equation (1) represents the vector momentum leaving the control volume, while the second integral gives the resultant force which the volume exerts on its bounding surface S.

A control volume with a bounding surface S generated by the line ABCDEFA can be constructed by referring to figure 1(a), which shows a cross section of a typical supersonic inlet operating at full capture mass flow. The line segments FA and CD are taken as perpendicular to the free-stream flow vector $\overline{V}_0$, while the segments ABC and ED generate the centerbody and cowl contours, respectively. The duct at station CD is assumed to have its walls parallel to $\overline{V}_0$, which corresponds to an inlet geometry in which the flow is returned to the axial direction after supersonic compression. 4574

Defining the vector $\hat{i}$ to be of unit magnitude and parallel to the free-stream flow $\overline{V}_0$ and taking the scalar product of $\hat{i}$ and equation (1) result in

$$\oint_S (\rho\overline{V} \cdot \hat{n})\overline{V} \cdot \hat{i}\, dA + \oint_S p\hat{n} \cdot \hat{i}\, dA = 0 \tag{2}$$

This equation is a scalar equation written for the axial component of momentum.

The upstream internal thrust exerted on the cowl is given by the pressure integral (the second integral in eq. (2)) over the surface generated by line ED, while the downstream force on the centerbody is given by the pressure integral over ABC. These two terms can be denoted by $-p_c^m A_c$ and $p_R^m A_R$, respectively, where p_c^m and p_R^m are the average effective pressures acting on A_c and A_R, the internal cowl projected area and the centerbody projected area, in that order.

With the previous notation, equation (2) may be written

$$p_R^m A_R + \int_{CD} (\rho\overline{V} \cdot \hat{n})\overline{V} \cdot \hat{i}\, dA + \int_{CD} p\hat{n} \cdot \hat{i}\, dA - p_c^m A_c - \rho_0 V_0^2 A_0 - p_0 A_0 = 0 \tag{3}$$

where $\int_{CD}$ indicates an integration over the surface generated by line CD. The evaluation of the two remaining integrals in equation (3) would require a detailed knowledge of the flow at the station CD. It is convenient then to consider the properties of the equivalent uniform flow having the same mass flow and momentum as the nonuniform flow at station CD. The term uniform here implies constant velocity and fluid properties across the duct as well as parallel flow, which at station CD is taken to be in the axial direction. By denoting the equivalent uniform flow with the subscript 2, equation (3) becomes

$$\rho_2 V_2^2 A_2 + p_2 A_2 - \rho_0 V_0^2 A_0 - p_0 A_0 + p_R^m A_R - p_c^m A_c = 0$$

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or, for a perfect gas,

$$P_2A_2G(M_2) = P_0A_0G(M_0) - p_R^m A_R + p_c^m A_c \tag{4}$$

The continuity equation written between FA and CD in terms of the D function is

$$P_2A_2D(M_2) = P_0A_0D(M_0) \tag{5}$$

Dividing equation (5) by equation (4) yields

$$N(M_2) = \frac{D(M_0)}{G(M_0) - \left(\frac{p_R^m}{P_0}\frac{A_R}{A_0} - \frac{p_c^m}{P_0}\frac{A_c}{A_0}\right)} \tag{6}$$

which determines $N(M_2)$ if p_R^m and p_c^m are known. To each value of the function N there are two corresponding Mach numbers, one supersonic and one subsonic. By taking the latter as pertinent to critical inlet operation since it evaluates the flow downstream of the terminal shocks, equation (6) then determines M_2, and, consequently, the effective recovery P_2/P_0 may be calculated from equation (5) as

$$\frac{P_2}{P_0} = \frac{A_0}{A_2}\frac{D(M_0)}{D(M_2)} \tag{7}$$

The functions G, D, and N are tabulated in reference 2.

Equations (6) and (7) cannot be evaluated unless the average pressures p_c^m and p_R^m are known. In general, because of the uncertainty in the location of the terminal shock, it is impossible to predict the internal cowl pressures with any assurance; however, for a zero-drag inlet, which for the purposes of this report will mean that $A_c = 0$, only a knowledge of the average pressure on the centerbody is necessary. In the event the centerbody is a two-dimensional ramp surface or a single cone with a sharp corner at the shoulder (as in fig. 1(b) in which BC is parallel to $\overline{V}_0$), the average centerbody pressure may be readily calculated by existing methods. It is assumed that all flow disturbances from the cowl lip strike the centerbody downstream of the shoulder B, a usual condition for the previously mentioned inlet types which are not overcontracted.

By referring to the zero-drag inlet geometry with a corner shoulder as the basic inlet, the effects of geometry changes, such as rounding the centerbody shoulder, may be determined by differentiating equation (7) with respect to φ, where φ is the bracketed term in the denominator of equation (6). Thus, the equation

$$\frac{d(P_2/P_0)}{d\varphi} = \frac{d(P_2/P_0)}{d\left[D(M_2)\right]} \frac{d\left[D(M_2)\right]}{d\left[N(M_2)\right]} \frac{d\left[N(M_2)\right]}{d\varphi} \tag{8}$$

gives

$$\frac{d(R/R_i)}{d(\varphi/\varphi_i)} = \left[1 - \frac{N(M_{2,i})}{N(M_0)}\right]\left[1 + \gamma(M_2)^2\right] \frac{G(M_{2,i})}{G(M_2)} \tag{9}$$

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where R denotes the total-pressure recovery P_2/P_0, and the subscript i refers to the basic inlet values of R, φ, and M_2. Thus, equation (9) evaluates the rate of increase of recovery with respect to φ. From equation (6) with $A_c = 0$, $N(M_0) < N(M_2)$. Thus, by equation (9), $d(R/R_i)/d(\varphi/\varphi_i) < 0$, which means that in order to obtain an increase in recovery $d(\varphi/\varphi_i)$ must be negative, or, equivalently, for a given A_R the mean ramp pressure p_R^m must decrease.

For most two-dimensional zero-drag inlets the turning loss may be calculated without considering the entire supersonic compression process. If the flow at the entrance station EB of figure 1(b) is uniform, the control volume enclosed by BCDEB may be considered in place of the former volume. In figure 1(c) the volume BCDEB is redrawn to larger scale with M_1 and α indicating the Mach number and flow inclination of the entrance flow with respect to the duct BC-ED. The specific type of supersonic compression that the free-stream flow M_0 undergoes to arrive at M_1 and α is immaterial.

The lip overhang angle η is a measure of the contraction ratio A_1/A_2:

$$\frac{A_1}{A_2} = \frac{\cos(\alpha - \eta)}{\cos \eta} \tag{10}$$

By use of equation (2) the momentum equation may be written as

$$P_2G(M_2) = P_1G(M_1) - 2P_1\left(\frac{q}{P}\right)_{M_1}\left[1 - \frac{\cos \alpha \cos(\alpha - \eta)}{\cos \eta}\right] \tag{11}$$

while the continuity equation is

$$P_2D(M_2) = P_1D(M_1) \frac{A_1}{A_2} = P_1D(M_1) \frac{\cos(\alpha - \eta)}{\cos \eta} \tag{12}$$

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Dividing equation (12) by equation (11) yields

$$N(M_2) = \frac{D(M_1)\cos(\alpha - \eta)}{\cos\eta\left\{G(M_1) - 2\left(\frac{q}{P}\right)_{M_1}\left[1 - \frac{\cos\alpha\cos(\alpha - \eta)}{\cos\eta}\right]\right\}} \tag{13}$$

Again for given M_1, α, and η, taking the subsonic root corresponding to $N(M_2)$ determines M_2 and thus $D(M_2)$ and gives for the recovery

$$\frac{P_2}{P_1} = \frac{\cos(\alpha - \eta)}{\cos\eta}\frac{D(M_1)}{D(M_2)} \tag{14}$$

For $\eta = \alpha/2$ no internal contraction occurs as can be seen from equation (10). Equations (13) and (14) then read

$$N(M_2) = \frac{D(M_1)}{G(M_1) - 2\left(\frac{q}{P}\right)_{M_1}(1 - \cos\alpha)} \tag{13a}$$

and

$$\frac{P_2}{P_1} = \frac{D(M_1)}{D(M_2)} \tag{14a}$$

The case $\eta = 0$, where the lip is directly over the corner shoulder B, also corresponds to the geometry of an unswept normal-shock inlet flying at M_1 and an angle of attack α. Equations (13) and (14) may also be taken as the equations for swept normal-shock inlets and are equivalent to those of reference 3 which considers such inlets.

RESULTS AND DISCUSSION

Significance of Effective Total Pressure

It is necessary at this point to consider the significance of the effective total-pressure recovery P_2/P_0 introduced in the previous section, ANALYSIS. Mathematically, P_2/P_0 was defined as the total-pressure recovery of a uniform stream having the same mass flow and total momentum at the inlet constant-area throat section as the actual nonuniform inlet flow. If the inlet were operating at its critical point and the constant-area throat section were infinitely long, the flow in this duct would eventually become a uniform subsonic stream if wall friction is neglected. The recovery of this stream would then correspond to the effective recovery computed herein and would take into account total-pressure losses due to mixing at constant area. The physical mechanism giving rise to what has been designated as a turning loss is thus a combination of internal shock and mixing losses.

Practical zero-drag inlet configurations will not provide sufficient constant-area throat length for complete mixing to occur. However, in general, the throat length should be sufficient to contain the terminal-shock system when the inlet is operating critically. If each filament of the subsonic flow leaving the throat is then assumed to diffuse isentropically, reference 4 indicates that the effective recovery after this diffusion process will be less than the initial effective recovery. Thus, the recoveries presented herein are an upper limit to the effective recovery that an engine at the terminus of an ideal subsonic diffuser would experience.

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It is desirable to establish that the effective recoveries of this report are an upper limit to the average total pressure of the flow at the exit of the subsonic diffuser independent of the subsonic diffuser configuration or performance. Reference 4 indicates that, for moderate distortion and effective duct Mach numbers up to 0.7, the mass flow weighted and effective recoveries are almost identical. As the mass flow weighted recovery of the flow leaving the inlet throat cannot increase, it follows that, under fairly general circumstances, the effective recovery at the throat is an upper limit to the inlet recovery for all subsonic diffusers.

Turning-Loss Considerations in Diffuser Design and Evaluation

The results of calculations made for single-cone zero-drag inlets with shock on lip and no internal contraction using equations (6) and (7) are presented in figure 2 by the solid lines which give effective recovery P_2/P_0 plotted against cone half-angle θ for various free-stream Mach numbers. For these inlets the throat area was set equal to the area projected normal to the average flow direction in the conical flow field at the entrance station. All curves are terminated at the lip detachment value; that is, for larger θ the conical shock is incapable of regular reflection at the cowl lip.

The dashed curves were taken from reference 5 and are included for comparison to illustrate the variation of inlet total-pressure recovery if it is assumed that the flow proceeds isentropically from the entrance station EB of figure 1(b) to station CD and there undergoes a normal shock. The lower recovery values given by equations (6) and (7) must then be attributed to the nonisentropic nature of the process involved in attaining a uniform axial flow after supersonic compression, in essence, a turning loss.

As may be seen from figure 2, the effect of turning losses favors smaller cone angles for zero-drag single-cone inlets than would otherwise be indicated. For example, at a Mach number of 4.0 the optimum cone half-angle is reduced to 25° as compared with 32° if turning losses are neglected.

As explained in the section entitled ANALYSIS, associating M_1 and α with the final ramp Mach number and angle permits a ready evaluation of the total-pressure recovery through the combined turning and terminal-shock process of a two-dimensional inlet.

Figures 3(a) and (b) are constructed from equations (13) and (14) for $\eta = \alpha/2$ and $\eta = \eta^{*}$, respectively, and indicate the extent to which increased turning influences recovery. At $\alpha = 0$ the value of P_2/P_1 is simply the normal-shock recovery for M_1 or M_1^{*}. All curves of figure 3(b) are terminated at the left for those values of α for which the lip reflected shock lies forward of the corner shoulder B. The over-all inlet recovery would be the product of the recovery P_2/P_1 given by equations (13) and (14) and P_1/P_0, which is the recovery through the supersonic compression process up to the entrance station.

The curves of figure 4 illustrate the variation in pressure recovery with Mach number for isentropic ramp and single-wedge zero-drag inlets. These inlets incorporated no internal contraction and were designed for maximum recovery consistent with full mass-flow capture. The calculations for the solid curves include turning losses, while for the dashed curves these losses were neglected. The decrement in pressure recovery for the isentropic inlet at Mach 4.0 is 17 percent. A similar calculation for a maximum contraction isentropic inlet gives a 19-percent decrease.

The performance of single-cone zero-drag inlets was readily evaluated for the corner-shoulder configuration as the average static pressure acting on the fore part of the centerbody was known. For a multicone or isentropic spike inlet, the average entrance station Mach number and the flow angle are usually known more accuately than the centerbody surface pressure. If this is true, equations (13) and (14), or, when appropriate, figures 3(a) or (b), may be employed to evaluate the effective recovery of such inlets using the average Mach number and flow angle to replace M_1 and α. The accuracy of such a procedure was checked for the case of single-cone inlets where, for example, at $M_0 = 3.0$ and $\theta = 26^{\circ}$ the exact turning loss from figure 2 is $0.054\ P_0$, while using the mean values of Mach number and flow angle in the conical field and equations (13) and (14) yields a loss of $0.052\ P_0$.

All consideration of turning losses has been previously confined to corner-shoulder configurations. The recovery will increase if the centerbody shoulder is rounded off. This is indicated by the discussion related to equation (9) where it was shown that p_R^m, the average pressure acting over the forward projected area of the ramp or centerbody, must decrease if the recovery is to increase. The extent to which the shoulder may be rounded, however, is limited by internal contraction considerations, as the rounding will increase the entrance area. Also, for extensive rounding, the reflected shock from the cowl may strike the centerbody before the shoulder turn is completed, a circumstance which would tend to increase p_R^m.

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An indication of the improvement in inlet performance that might be expected because of rounding is given by the following examples. A Mach 4.0, isentropic ramp inlet with the corner shoulder rounded off to a circular arc as illustrated in figure 1(d) and with $A_s/A_R = 0.1$ has a calculated increase in total-pressure recovery of $0.01\ P_0$ compared with the corner-shoulder configuration. This is equivalent to reducing the turning loss from the original $0.09\ P_0$ of figure 4 to $0.08\ P_0$. The same modification at $M_0 = 3.0$ also resulted in only a $0.01\ P_0$ increase in recovery. In both examples the pressures on the shoulder were computed by Prandtl-Meyer theory.

The practical use of the momentum-continuity method for calculating inlet recovery is not limited to the case of a uniform flow at the entrance station or to shock-on-lip zero-drag inlets. For an inlet operating at above design speed the compression shock or shocks may fall downstream of the cowl lip. For zero-drag two-dimensional inlets the ramp pressures may still be calculated, and, thus, equations (6) and (7) may be used to calculate the recovery. For either the two-dimensional or axisymmetric inlet the pressure recovery will be independent of centerbody translation as long as the inlet neither spills flow nor overcontracts.

Blockage Considerations

The function N appearing in equation (6) assumes a maximum value of 0.4564 at Mach 1.0 for $\gamma = 1.4$ and decreases monotomically in value on either side of Mach 1.0. Quantitatively, the condition that the function N yield a meaningful result may be formulated by writing equation (6), with $A_c = 0$, as

$$\frac{p_R^m}{p_0}\frac{A_R}{A_0} \leq \left[G(M_0) - \frac{D(M_0)}{N(M = 1.0)}\right]\left(\frac{P}{p}\right)_{M_0} \equiv Z \tag{15}$$

A plot of the right side of equation (15), designated Z, for $\gamma = 1.4$ is given in figure 5.

A particular inlet geometry may be such that equation (6) yields values of N greater than $N(M = 1.0)$. If this should happen, the interpretation is that no equivalent flow M_2 exists which would satisfy the inlet geometry; that is, the assumed flow structure is untenable and, as a result, the inlet will not operate at full capture mass flow.

In order for an inlet of the type in figure 1(b) to operate at full capture mass flow, the centerbody force parameter $\frac{p_R^m}{p_0} \frac{A_R}{A_0}$ must be less than the critical value Z. For a given p_R^m/p_0 this requirement then places a lower limit on the over-all contraction ratio for the inlet $A_2/A_0 = 1 - (A_R/A_0)$. The use of variable-geometry techniques would be unsuccessful in circumventing this limitation on over-all contraction since the critical force parameter may not be exceeded at any time subsequent to starting the inlet.

If the inlet centerbody is considered as a wind tunnel model and the straight inlet cowling as wind tunnel walls, the condition specified by equation (15) and figure 5 determines whether the tunnel may start. This wind tunnel starting criterion supplements the usual Kantrowitz condition in that both criteria must be satisfied.

Usually those configurations satisfying the Kantrowitz requirements will also satisfy the momentum considerations of equation (15). There are exceptions to this rule, however. For example, a normal-shock inlet operating at zero mass flow was calculated to block a wind tunnel if sized according to Kantrowitz for all Mach numbers up to 4.0, the range of calculations made. For this case the pressure p_R^m is the pitot pressure, and equation (15) requires approximately a 20-percent reduction in model size to permit starting.

Whenever the starting criterion presented herein is more stringent than the Kantrowitz requirement, that is, requires a smaller model size, it means in essence that the wind tunnel model, if sized according to Kantrowitz, introduces greater total-pressure losses in the tunnel flow than a normal shock spanning the entire test section. For instance, in the case of the zero-flow normal-shock inlet described previously, the loss in total pressure suffered by the tunnel flow because of the detached bow shock, the shocks following the reexpansion around the inlet lip, and whatever mixing occurs exceed those of a normal shock at the tunnel Mach number.

CONCLUDING REMARKS

For zero-drag external-compression inlets a loss in total pressure is inherent in turning the flow back to the axial direction after supersonic compression. An analysis based on momentum and continuity considerations, applicable to most zero-drag inlets, indicates that these losses may at times become as high as 20 percent of the inlet recovery at Mach 4.0. For zero-drag single-cone inlets, the effect of including turning losses decreases the cone angle required for optimum pressure recovery.

The momentum-continuity analysis applied to the evaluation of inlet turning losses yields a wind tunnel starting criterion which supplements the usual Kantrowitz condition and is particularly appropriate for the testing of high-drag models.

Lewis Flight Propulsion Laboratory
National Advisory Committee for Aeronautics
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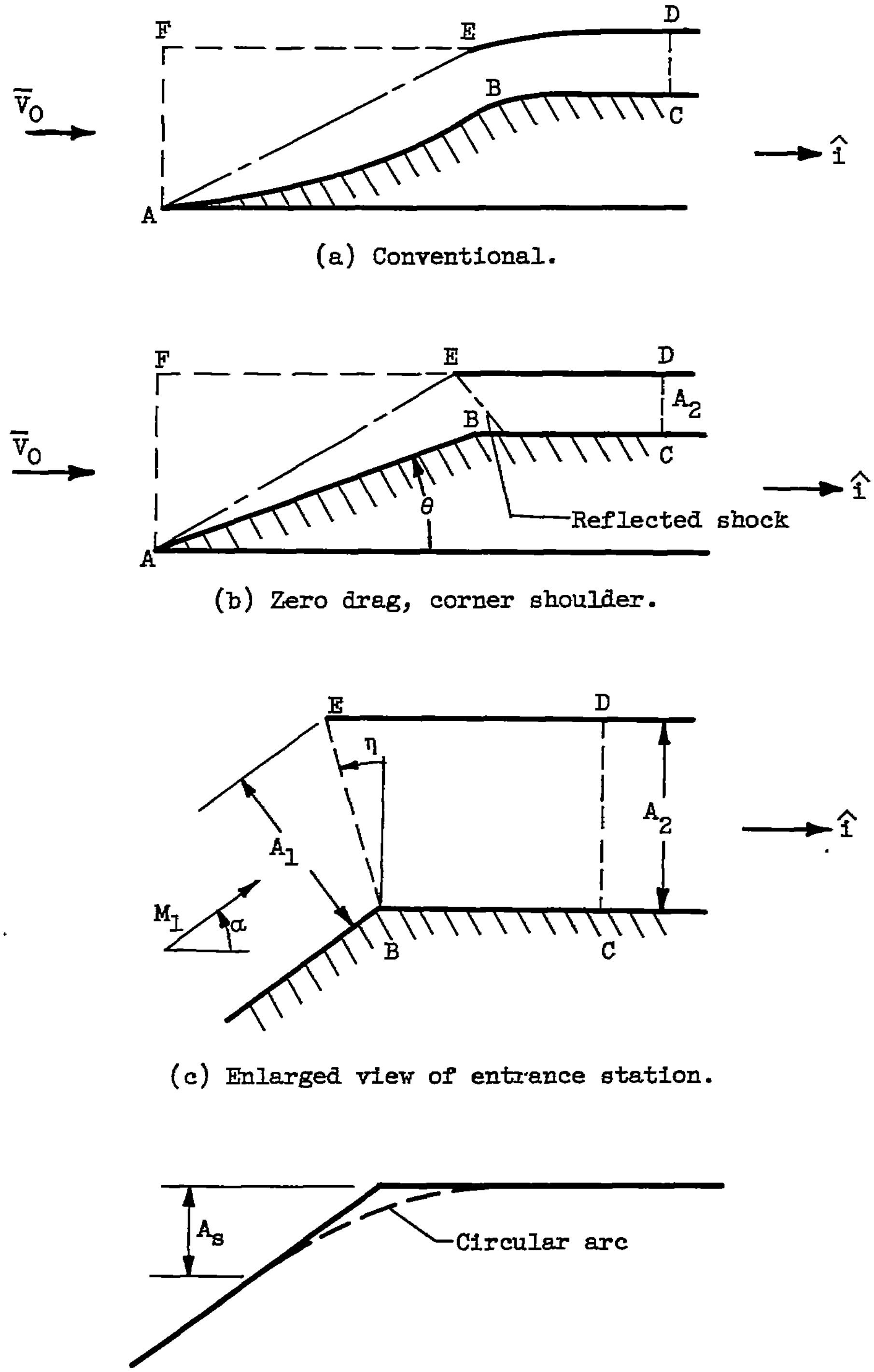

(a) Conventional.

(b) Zero drag, corner shoulder.

(c) Enlarged view of entrance station.

(d) Detail of shoulder rounding.

Figure 1. - Schematic diagram of inlet geometries.

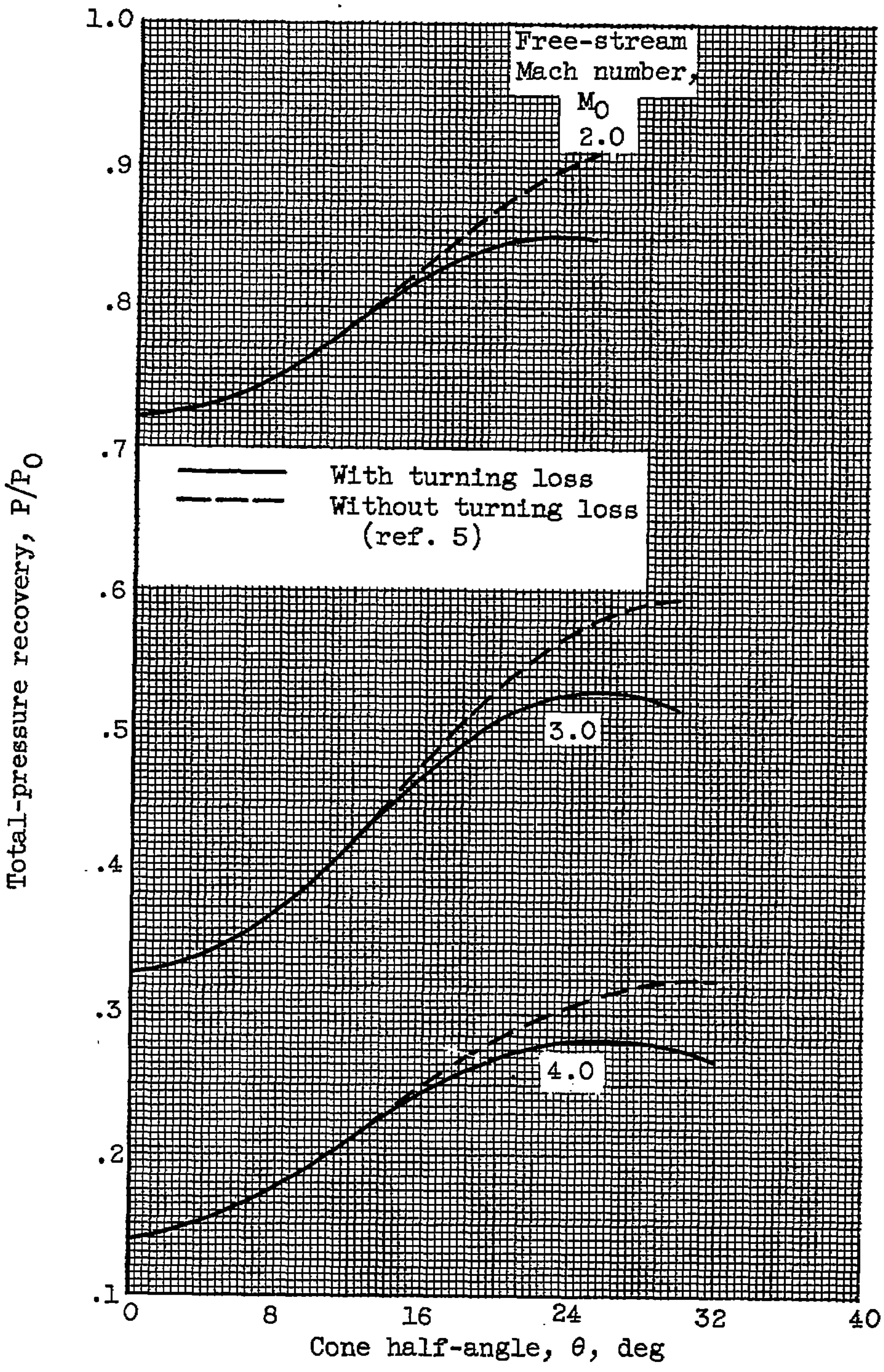

Figure 2. - Performance of single-cone inlets with no internal contraction.

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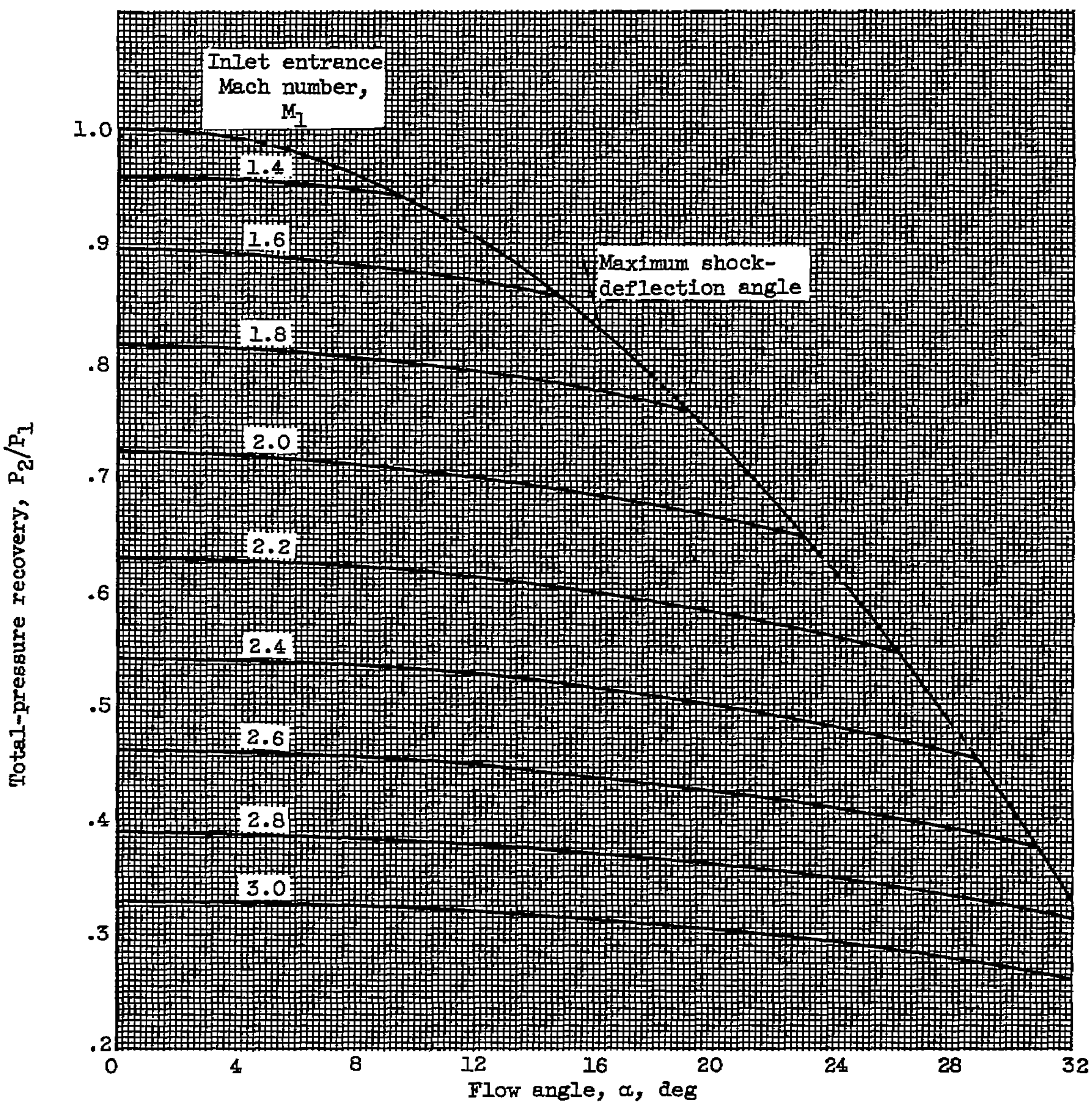

(a) No internal contraction.

Figure 3. - Total-pressure recovery for two-dimensional turns.

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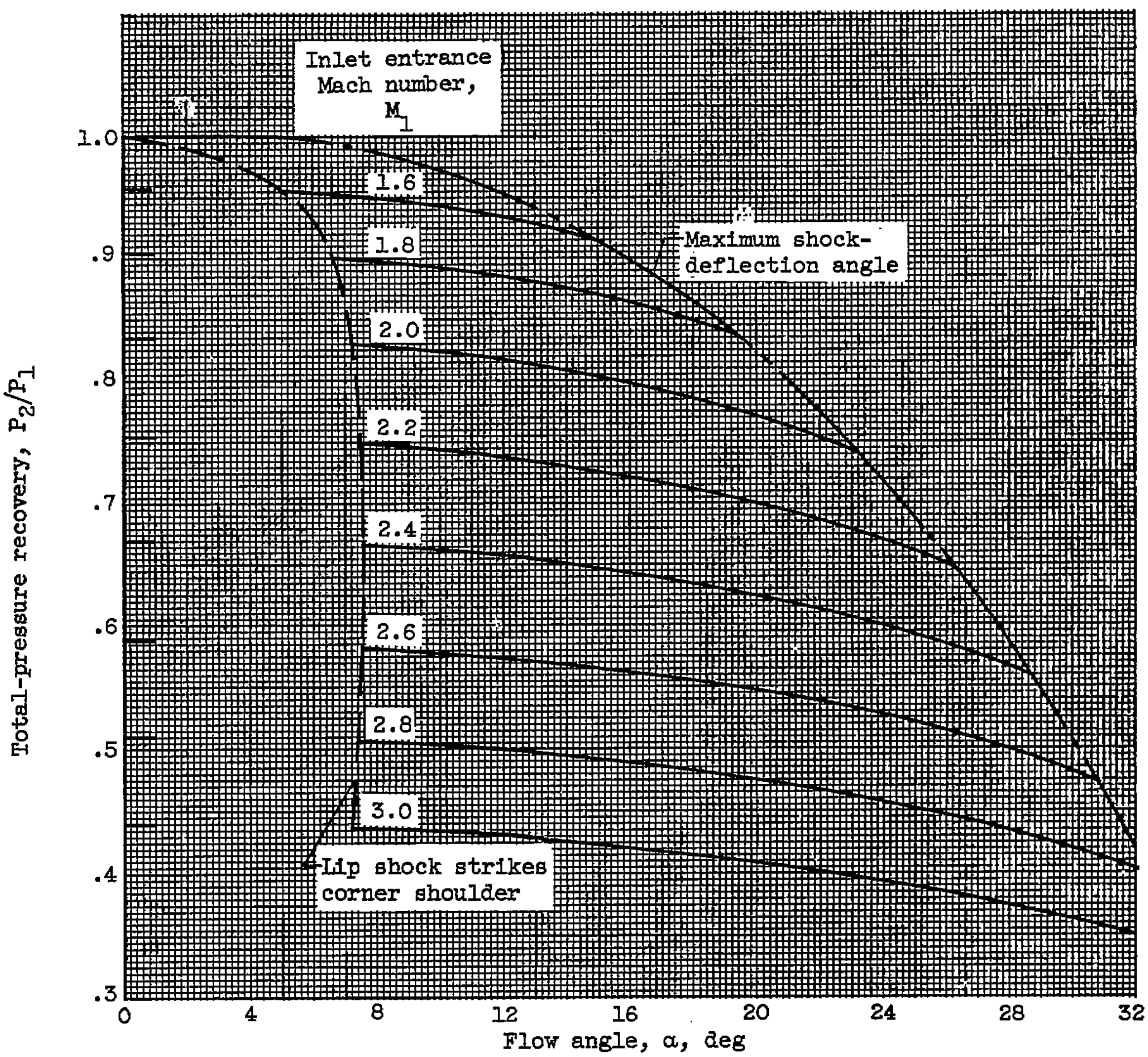

(b) Maximum internal contraction.

Figure 3. - Concluded. Total-pressure recovery for two-dimensional turns.

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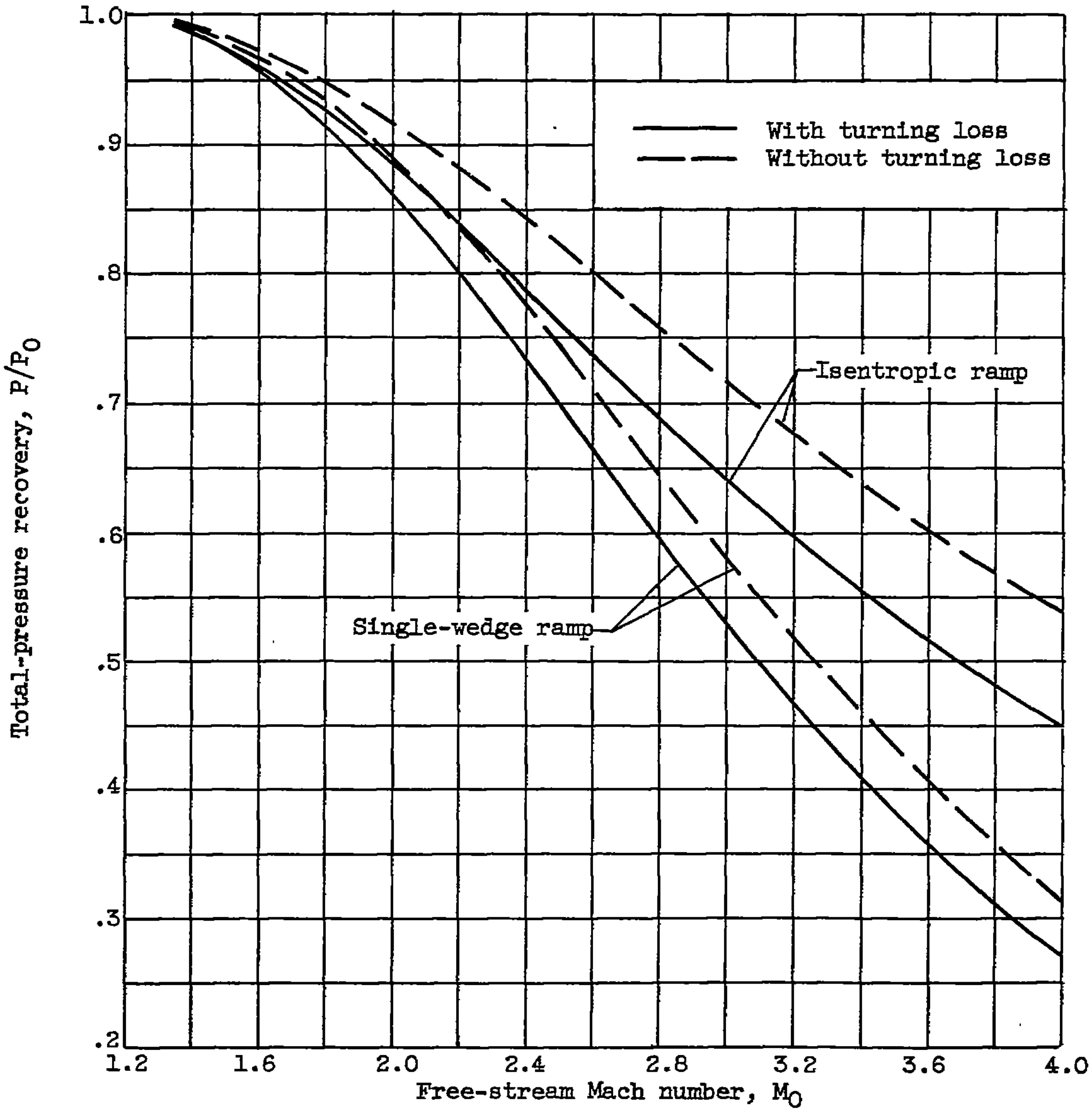

Figure 4. - Performance of optimum isentropic ramp and single-wedge zero-drag inlets.

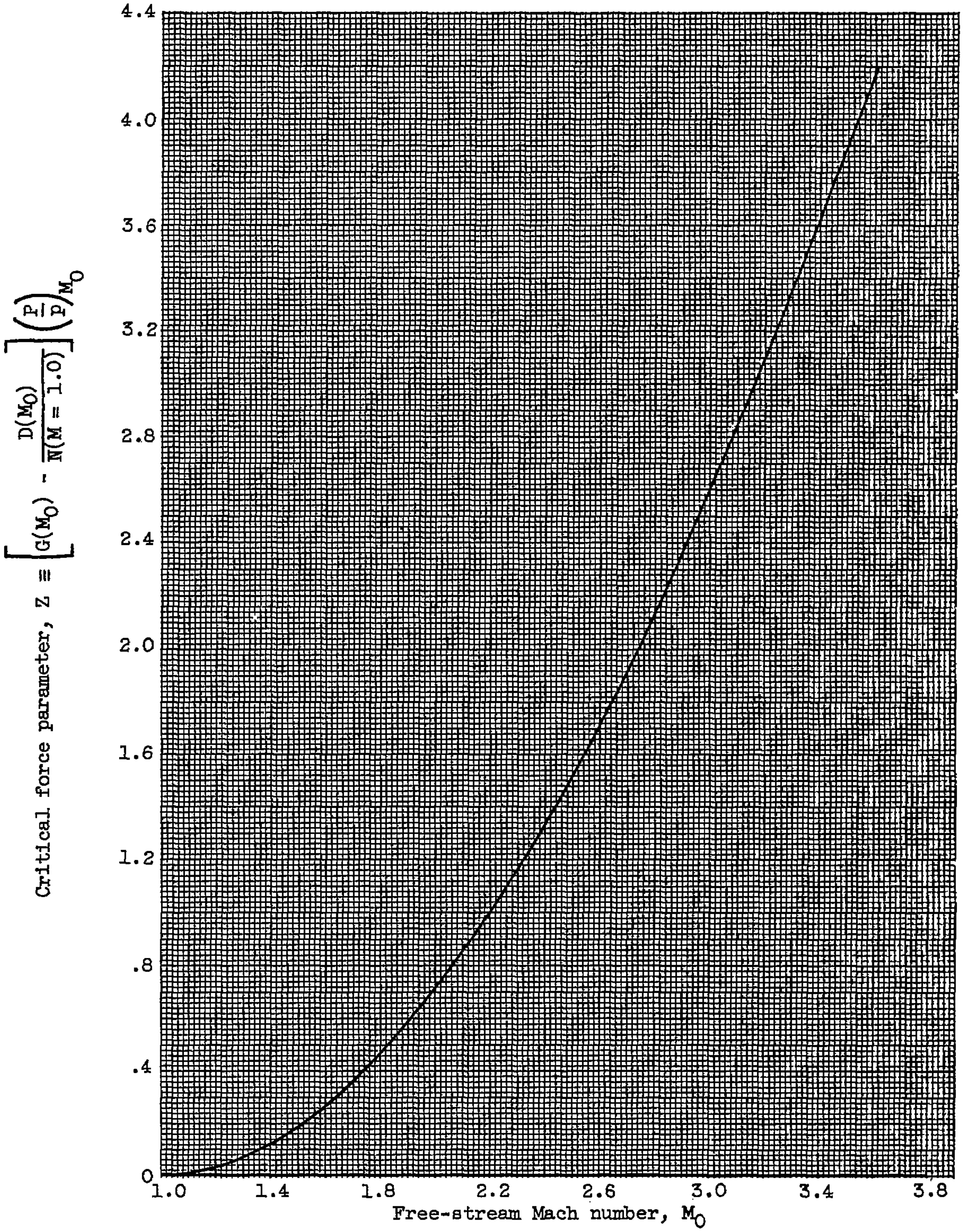

Figure 5. - Wind tunnel starting criterion.

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